

SU(3) String-Flip Potential Models and Nuclear Matter

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ABSTRACT

For over 50 years attempts have been made to explain the properties of nuclear matter in terms of constituent nucleons with very little success. Here we will investigate one class of many possible models, string-flip potential models, in which flux-tubes are connected between quarks (in a gas/plasma) to give a minimal overall field configuration. A general overview of the current status of these models, along with some of our recently finished work, shall be given. It shall be shown that these models seem promising in that they do get most of the bulk properties of nuclear matter correct with the exception of nuclear binding. Finally we will conclude with a brief discussion on ways to improve the string-flip potential models in an attempt to obtain nuclear binding (currently we are investigating short range one-gluon exchange effects – some preliminary results shall be mentioned).

1. Overview

The main objective of our work is to attempt to describe nuclear matter in terms of its constituent quarks. A difficult task indeed, for over the past 50 years many attempts have been made with very little success. The main difficulty is due to the nonperturbative nature of QCD. The most rigorous method for handling multiquark systems to date is lattice QCD, but given the magnitude of our problem it appears unlikely to be useful in the near future, due to its computationally intensive nature. As a result we must consider more phenomenological means.

The basic idea here is to construct models which are motivated by lattice QCD theory and nucleon based models of nuclear matter. A very crude model should be able to get most of bulk properties of nuclear matter correct: i.e.

- Nucleon gas at low densities with no van der Waals forces.
- Nucleon binding at higher densities.
- Nucleon swelling and saturation of nuclear forces with increasing density.
- Quark-gluon plasma at extremely high densities.

There are many models out there that attempt to fill this shopping list but we have found none that covers it completely.

*Talk presented by M. Boyce.

Here, we shall restrict ourselves to a particular class of models, called string-flip Potential Models,^{1,4,8} which fills all the items on our list with the exception of nucleon binding. The later part of our discussion shall be a way of perhaps improving these models in an attempt to get nucleon binding.

2. String-Flip Potential Model

String-flip potential models are models which are inspired by lattice QCD in that they attempt to mimic the flux-tube dynamics. In the most general setting, one assumes that the quarks, in question, move slowly enough that their fields have enough time to reconfigure themselves in order to minimize the overall potential energy: i.e.

$$V = \min\left\{ \sum_{\{q_m \dots q_n\}} v(\vec{r}_m \dots \vec{r}_n) \mid \bigcup_{\{m \dots n\}}^{\sim} \{q_m \dots q_n\} = \{q_1 \dots q_{N_q}\} \right\}, \quad (1)$$

where the N_q quarks are placed in a cube of side L and subjected to periodic boundary conditions, to simulate continuous quark matter. The sum is over all gauge invariant sets $\{q_m \dots q_n\}$ of quarks, such that at least one element from each set lies inside a common box, whose disjoint union, \bigcup^{\sim} , makes up the complete colour singlet set $\{q_1 \dots q_{N_q}\}$ of N_q quarks. It is easy to see that this potential allows for complete minimal quark clustering separability at low densities without suffering from van der Waals forces.

At present these models are quite crude in that they do not include short range one-gluon exchange phenomena and spin effects, and are flavour degenerate.

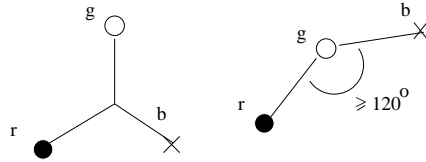
2.1. $SU_\ell(3)$ & $SU_h(3)$ Models

To simplify matters we shall assume that the multiquark potential, eq. (1), runs over triplets of quarks and that the colour is fixed to a given quark: i.e.

$$V = \min\left\{ \sum_{\{q_r q_g q_b\}} v(\vec{r}_r, \vec{r}_g, \vec{r}_b) \mid \bigcup_{\{rgb\}}^{\sim} \{q_r q_g q_b\} = \{q_1 \dots q_{N_q}\} \right\}. \quad (2)$$

For a linear model, $SU_\ell(3)$, the three body potential, $v(\vec{r}_r, \vec{r}_g, \vec{r}_b)$, is given by²

$$v_\ell(\vec{r}_r, \vec{r}_g, \vec{r}_b) = \sigma \begin{cases} r_{br} + r_{rg}; & \text{if } \angle brg \geq 120^\circ \\ r_{rg} + r_{gb}; & \text{if } \angle rgb \geq 120^\circ \\ r_{gb} + r_{br}; & \text{if } \angle gbr \geq 120^\circ \\ \sqrt{\frac{3}{2}\xi_{rgb}^2} + \frac{\sqrt{3}}{2}A; & \text{otherwise} \end{cases} \quad (3)$$



where $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$, $\xi_{rgb}^2 = (r_{rg}^2 + r_{gb}^2 + r_{br}^2)/3$, and A is the area enclosed by the triangle $\triangle rgb$.

For a harmonic oscillator model, $SU_h(3)$, our three body potential is simply

$$v_h(\vec{r}_r, \vec{r}_g, \vec{r}_b) = \frac{1}{2} k \xi_{rgb}^2 \quad \begin{array}{c} \circ \\ \diagup \quad \diagdown \\ \bullet \quad \bullet \end{array} \quad (4)$$

where we have assumed quarks of equal mass (*cf. ref. 1*).

The linear model is inspired by lattice QCD whereas the harmonic oscillator is used because it has some nice analytical properties which allows us to make a consistency check.

2.2. Computation

Here we use a variational procedure to find the binding energy per nucleon, E_B ($= \bar{E}(\rho) - \bar{E}(0)$), and the correlation length, β , as a function of nucleon density, ρ .

The variational wave function,

$$\Psi_{\alpha\beta\rho} = \chi_{\text{Correlation}} \Phi_{\text{Slater}} , \quad (5)$$

is a trial function which consists of the product of a symmetric correlation piece,

$$\chi_{\text{Correlation}} = e^{-\frac{1}{2} \sum_{\{rgb\}} (\beta \xi_{rgb})^\alpha} , \quad (6)$$

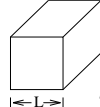
Correlation length $\beta \sim \frac{1}{r_{\text{rms}}}$.

Variational parameter $\alpha = \begin{cases} 1.75 & \text{if } SU_\ell(3) \\ 2 & \text{if } SU_h(3) \end{cases}$ and is found by minimizing $E(\rho = 0)$.

\sum over the set of triplets $\{rgb\}$ which minimizes V .

and an antisymmetric Fermi piece,

$$\Phi_{\text{Slater}} = \phi_r(\rho) \phi_g(\rho) \phi_b(\rho) . \quad (7)$$

 Density $\rho = \frac{\text{Nucleons}}{L^3}$

Product of Slater determinants containing

$$\phi_i(\vec{r}_j) = \sin\left(\frac{2\pi}{L} \vec{n}_i \cdot \vec{r}_j + \delta_i\right) ,$$

where $\delta_i = 0$ or $\pi/2$, and $(\vec{n}_i)_{a(=x,y,z)} = 0, \pm 1, \pm 2, \dots$

This gives a smooth interpolation between a correlated (nucleon) gas at low densities and an uncorrelated (Fermi) gas at high densities.

The results of the variational procedure are shown in Fig. 1. The solid lines show the minimal $E_B(\rho)$ and $\beta(\rho)$ curves, where we have minimized $\bar{E}(\rho, \beta)$. The integrals involved in evaluating $\bar{E}(\rho, \beta)$ were done using the Metropolis algorithm^{5,3} along with a bag of tricks to minimize cpu time.¹

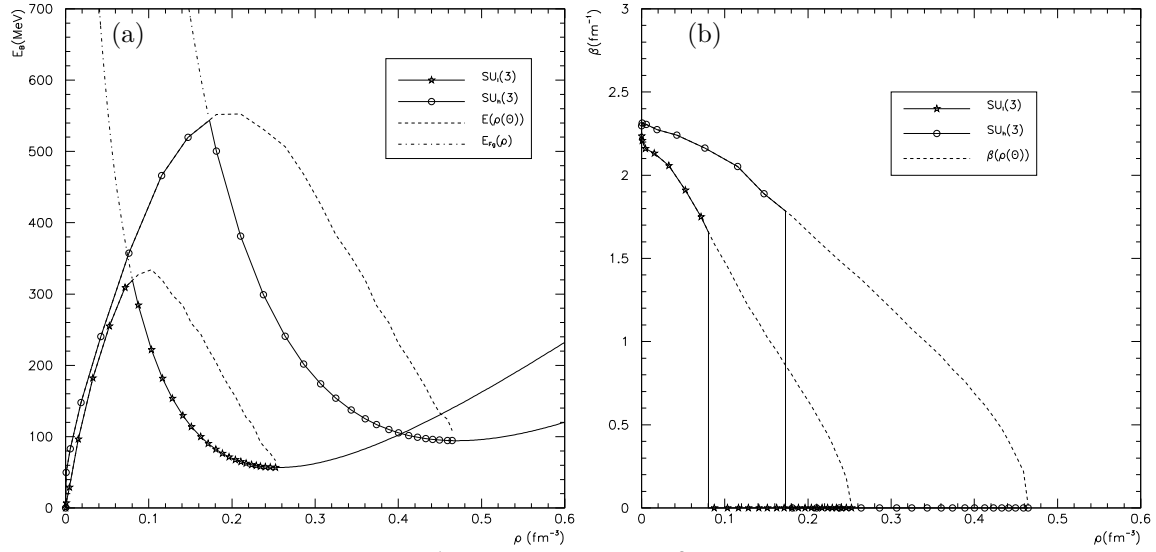


Fig. 1: $E_B(\text{MeV})$, (a), and $\beta(\text{fm}^{-1})$, (b), versus $\rho(\text{fm}^{-3})$ for $SU_\ell(3)$ (\star) and $SU_h(3)$ (\circ), where $m_q = 330\text{MeV}/c^2$, $\sigma = 0.91\text{GeV}/\text{fm}$ and $k \approx 3244\text{MeV}/\text{fm}^2$.¹ The dashed curves represent the remnants of the minimal $E_B(\rho, 0)$ and $E_B(\rho, \beta)$ trajectories.

From Fig. 1.a we see that there is an overall saturation of nucleon forces followed by a transition (the kink in the solid line) to a Fermi gas, but no nucleon binding. Fig. 1.b illustrates an overall swelling of nucleons with increasing ρ until they become infinite beyond the transition point.

3. The Show So Far!

We have demonstrated that string-flip potential models gets the overall bulk properties of nuclear matter correct with the exception of binding.

So what can we do to obtain binding?

- a) What about allowing the colour to move around?
 - This seems unlikely; it has been investigated in an $SU_h(2)$ model by HP.⁴
- b) What about allowing higher order flux-tube topologies?
 - This seems unlikely; it been investigated in an – albeit slightly ill[†] – $SU_h(3)$ chain model by HP.⁴
- c) What about “a)” and “b)”?
 - This seems unlikely; it has been investigated by many six quark (non-string-flip) models^{7,6} which suggests six quark states like to dissociate into two nucleons.
- d) What about including short range forces?
 - This seems likely; we are currently investigating this possibility.

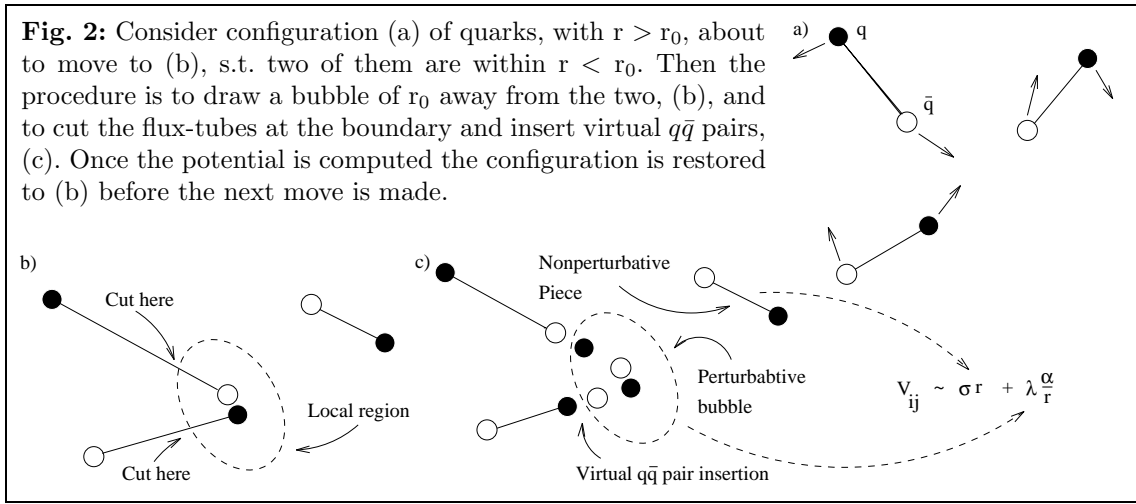
[†]Predicts nucleon shrinking.¹

3.1. Short Range Forces

At the moment we are investigating the inclusion of colour Coulomb effects. The difficulty arises here when trying to combine perturbative and nonperturbative fields in a consistent fashion. To simplify our study we consider an $SU_\ell(2)$ potential model, which looks a lot like $SU_\ell(3)$ (*cf. ref. 8*), for $q\bar{q}$ pairs: i.e.

$$v_{ij} \sim \begin{cases} \sigma(r_{ij} - r_0) & \text{if } r_{ij} > r_0 \\ \alpha_s \lambda_{ij} \left(\frac{1}{r_{ij}} - \frac{1}{r_0} \right) & \text{if } r_{ij} < r_0 \end{cases} \quad \begin{array}{c} \text{Linear} \\ \text{Coulomb} \end{array} \quad (8)$$

where $\lambda_{ij} = -3/4, 1/4$ for unlike and like colours respectively, and $\alpha_s = 0.1$. Fig. 2 illustrates the dynamics of the model.



Notice that this model allows us to construct colourless objects because of the insertion the virtual $q\bar{q}$ pairs. These virtual quarks are used as a tool to get the overall length of flux-tube correct. They are not used in computing the Coulomb term however, as the field energy is already taken into account by the “real” quarks in the bubble. In general, once the bubbles have been determined, one must reconfigure the flux-tubes in order to minimize the linear part of the potential.

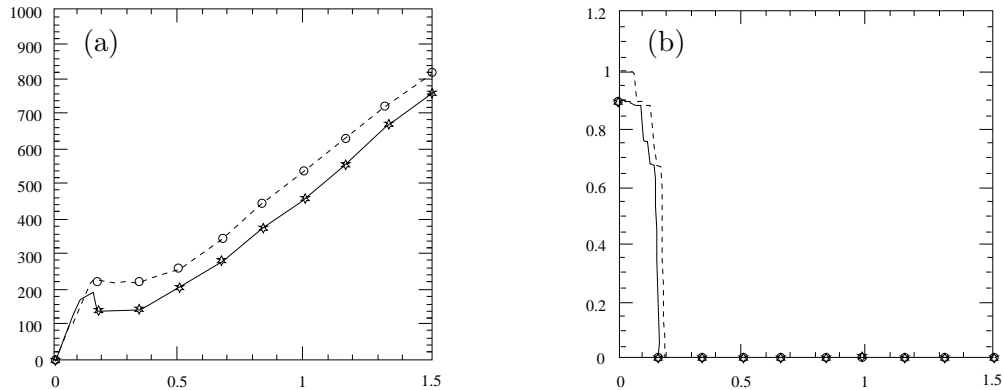


Fig. 3: $E_B(\rho)$, (a), and $\beta(\rho)$, (b), for $r_0 = 0$ fm (—) and $r_0 = 0.1$ fm (—).

Some preliminary results of these models are shown in Fig. 3. For $r_0 = 0$ fm we get back the same results – as expected – and for $r_0 = 0.1$ fm we get a slightly different answer.

Although the model is currently for $SU_\ell(2)$ it should be easy to extend it to a full $SU_\ell(3)$ model with all the one-gluon exchange phenomena. The results presented here are very preliminary as our wave function is slightly sick. By this we mean that we used our old wave function, eq. (5), where the correlation piece was over the modified flux-tubes and a new piece was thrown in to take care of the local Coulomb interactions as they occurred: i.e.

$$\Psi_{\alpha\alpha'\beta\rho} \sim \chi_{\alpha\beta}^{(\text{Linear})} \times \chi_{\alpha'\beta}^{(\text{Coulomb})} \times \Phi_\rho^{(\text{Fermi})} . \quad (9)$$

However because the flux-tubes and bubbles can now be created or destroyed the wave function is, in general, no longer smooth and continuous: i.e. we are not guaranteed a variational lower bound. Currently we are working to rectify the matter and hope to have some results out in the near future.

4. Closing Remarks

We have demonstrated that string-flip potential models appear to be fairly good candidates for explaining nuclear matter at the constituent quark level. Crude models seem to get most of the bulk properties with the exception of nuclear binding. We have also shown that it is fairly reasonable to assume that these models must be extended if we are ever going to achieve binding. In this light we have modified the model to include one-gluon exchange effects. Currently it only includes Coulomb effects, which we hope to have results for sometime soon. Further down the road we would like to include the rest of the one-gluon exchange phenomena.

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